Limits of Trig Functions

1. $\lim_{x\to 0}\frac{\sin^3 x}{(2x)^3}=\lim_{x\to 0}\frac{\sin^3 x}{8x^3}=\lim_{x\to 0}\left(\frac{1}{8}\cdot\frac{\sin x}{x}\cdot\frac{\sin x}{x}\cdot\frac{\sin x}{x}\right)=\frac{1}{8}\cdot 1\cdot 1\cdot 1=\frac{1}{8}$

2. $\lim_{x\to 0}\frac{\sin x}{\sqrt[3]{x}}=\lim_{x\to 0}\left(\frac{\sin x}{x^{\frac{1}{3}}}\cdot\frac{x^{\frac{2}{3}}}{x^{\frac{2}{3}}}\right)=\lim_{x\to 0}\left(\frac{\sin x}{x}\cdot x^{\frac{2}{3}}\right)=1\cdot 0=0$

3. $\lim_{x\to 0}\left(\frac{3x+\sin x}{x}\right)=\lim_{x\to 0}\left(\frac{3x}{x}+\frac{\sin x}{x}\right)=\lim_{x\to 0}\left(3+\frac{\sin x}{x}\right)=3+1=4$

4. $\lim_{x\to 0}\frac{2+\sin x}{3+x}=\lim_{x\to 0}\frac{2+\sin 0}{3+0}=\frac{2+0}{3+0}=\frac{2}{3}$

5. $\lim_{x\to 0}\frac{2\cos x-2}{3x}=\lim_{x\to 0}\frac{2(\cos x-1)}{3x}=\lim_{x\to 0}\left(\frac{2}{3}\cdot\frac{(\cos x-1)}{x}\right)=\frac{2}{3}\cdot 0=0$

6. $\lim_{x\to 0}\frac{\sin(-3x)}{4x}=\lim_{x\to 0}\frac{\sin(-3x)}{4x}\cdot\frac{\frac{-3}{4}}{\frac{-3}{4}}=-\frac{3}{4}\lim_{x\to 0}\frac{\sin(-3x)}{-3x}=-\frac{3}{4}\cdot 1=-\frac{3}{4}$

7. $\lim_{x\to 0}\frac{4x^2+3x\sin x}{x^2}=\lim_{x\to 0}\left(\frac{4x^2}{x^2}+\frac{3x}{x}\cdot\frac{\sin x}{x}\right)=\lim_{x\to 0}\left(4+3\cdot\frac{\sin x}{x}\right)=4+3=7$

8. $\lim_{x\to 0}\frac{\cos x}{1-\sin x}=\lim_{x\to 0}\left(\frac{\cos 0}{1-\sin 0}\right)=\frac{1}{1-0}=1$

9. $\lim_{x\to 0}\frac{1-\cos 3x}{x}=\lim_{x\to 0}\frac{1-\cos 3x}{x}\cdot\frac{3}{3}=3\lim_{x\to 0}\left(\frac{1-\cos 3x}{3x}\right)=3\cdot 0=0$

10. $\lim_{x\to 0}\frac{x\sin x}{x^2+1}=\lim_{x\to 0}\left(\frac{0\cdot\sin 0}{0^2+1}\right)=\frac{0}{1}=0$

11. $\lim_{x\to 0}\frac{1-2x^2-2\cos x+\cos^2 x}{x^2}=\lim_{x\to 0}\frac{-2x^2+1-2\cos x+\cos^2 x}{x^2}=\lim_{x\to 0}\frac{-2x^2+\left(1-2\cos x+\cos^2 x\right)}{x^2}=$
$\lim_{x\to 0}\frac{-2x^2}{x^2}+\frac{(1-\cos x)(1-\cos x)}{x^2}=\lim_{x\to 0}\frac{-2x^2}{x^2}+\frac{(1-\cos x)}{x}\frac{(1-\cos x)}{x}=\lim_{x\to 0}-2+\frac{(1-\cos x)}{x}\frac{(1-\cos x)}{x}=$
$-2+0\cdot 0=-2$

12. $\lim_{x\to 0}\frac{1-\cos x}{\sin x}=\lim_{x\to 0}\frac{(1-\cos x)}{\sin x}\cdot\frac{(1+\cos x)}{(1+\cos x)}=\lim_{x\to 0}\frac{1-\cos^2 x}{\sin x(1+\cos x)}=\lim_{x\to 0}\frac{\sin^2 x}{\sin x(1+\cos x)}=$
$\lim_{x\to 0}\frac{\sin x}{(1+\cos x)}=\lim_{x\to 0}\frac{\sin 0}{(1+\cos 0)}=\frac{0}{1+1}=0$

13. $$\lim_{x\to 0}\frac{x+\tan x}{\sin x}=\lim_{x\to 0}\frac{x+\dfrac{\sin x}{\cos x}}{\sin x}=\lim_{x\to 0}\frac{\dfrac{x\cos x+\sin x}{\cos x}}{\sin x}=\lim_{x\to 0}\frac{x\cos x+\sin x}{\sin x\cos x}=$$
$$\lim_{x\to 0}\frac{x\cos x}{\sin x\cos x}+\frac{\sin x}{\sin x\cos x}=\lim_{x\to 0}\left(\frac{x}{\sin x}+\frac{1}{\cos x}\right)=1+1=2$$

14. $$\lim_{x\to 0}x\cot x=\lim_{x\to 0}x\frac{\cos x}{\sin x}=\lim_{x\to 0}\left(\frac{x}{\sin x}\cdot\cos x\right)=1\cdot 1=1$$

15. $$\lim_{x\to 0}\frac{\sin^2 x}{x^2}=\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\sin x}{x}=1\cdot 1=1$$

16.
$$\lim_{x\to 0}\frac{\csc 2x}{\cot x}=\lim_{x\to 0}\frac{\tan x}{\sin 2x}=\lim_{x\to 0}\frac{\dfrac{\sin x}{\cos x}}{2\sin x\cos x}=\lim_{x\to 0}\frac{\sin x}{2\sin x\cos^2 x}=\lim_{x\to 0}\frac{1}{2\cos^2 x}=\frac{1}{2\cdot 1}=\frac{1}{2}$$